

# Traversing the QCD Phase Transition: Quenching Out of Equilibrium vs. Slowing Out of Equilibrium vs. Bubbling Out of Equilibrium

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I review arguments for the existence of a critical point  $E$  in the QCD phase diagram as a function of temperature  $T$  and baryon chemical potential  $\mu$ . I describe how heavy ion collision experiments at the SPS and RHIC can discover the tell-tale signatures of such a critical point, thus mapping this region of the QCD phase diagram. I contrast the different ways in which the matter produced in a heavy ion collision can be driven out of equilibrium: quenching out of equilibrium (possible, but not guaranteed, if the transition region is traversed at  $\mu \ll \mu_E$ ) vs. slowing out of equilibrium (guaranteed for  $\mu \sim \mu_E$ ) vs. bubbling out of equilibrium (possible, but not guaranteed, for  $\mu \gg \mu_E$ ). Quenching or bubbling create and amplify distinct, detectable, non-gaussian fluctuations. In contrast, slowing out of equilibrium reduces the magnitude of the specific, detectable, gaussian fluctuations which signal the presence of the critical point.

## 1. The Critical Point

One goal of relativistic heavy ion collision experiments is to explore and map the QCD phase diagram as a function of temperature and baryon chemical potential. Recent theoretical developments suggest that a key qualitative feature, namely a critical point which in a sense defines the landscape to be mapped, may be within reach of discovery and analysis by the CERN SPS or by RHIC, as data is taken at several different energies [1,2]. The discovery of the critical point would in a stroke transform the map of the QCD phase diagram from one based only on reasonable inference from universality, lattice gauge theory and models into one with a solid experimental basis [3].

In QCD with two massless quarks ( $m_{u,d} = 0$ ;  $m_s = \infty$ ) the phase transition at which chiral symmetry is restored as  $T$  is increased with  $\mu = 0$  is likely second order and belongs to the universality class of  $O(4)$  spin models in three dimensions [4]. Below  $T_c$ , chiral symmetry is broken and there are three massless pions. At  $T = T_c$ , there are four massless degrees of freedom: the pions and the sigma. Above  $T = T_c$ , the pion and sigma correlation lengths are degenerate and finite.

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\*Many thanks to the organizers for a stimulating conference in Adelaide. I am also grateful to the Department of Energy's Institute for Nuclear Theory at the University of Washington for generous hospitality and support during the writing of this contribution. Research supported in part by a DOE OJI Award, by the A. P. Sloan Foundation and by the DOE under agreement DE-FC02-94ER40818. Preprint MIT-CTP-2979.

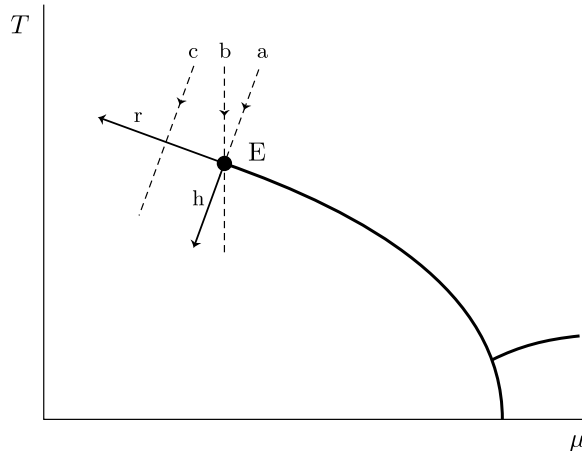


Figure 1. Sketch of the QCD phase diagram as a function of temperature  $T$  and baryon chemical potential  $\mu$ . Chiral symmetry is broken at low  $T$  and  $\mu$ . As  $T$  is increased, chiral symmetry is approximately restored via a smooth crossover to the left of  $E$  or a first order phase transition to the right of  $E$ . The symmetry is only approximately restored because the light quarks are not massless. At the critical point  $E$  at which the line of first order phase transitions ends, the transition is second order and is in the Ising universality class. (At large  $\mu$  and small  $T$ , there are color superconducting phases.) The Ising model  $r$ -axis and  $h$ -axis and the trajectories  $a$ ,  $b$  and  $c$  will be discussed below.

In nature, the light quarks are not massless. Because of this explicit chiral symmetry breaking, the second order phase transition is replaced by an analytical crossover: physics changes dramatically but smoothly in the crossover region, and no correlation length diverges. This picture is consistent with present lattice simulations [5], which suggest  $T_c \sim 140 - 190$  MeV [6].

Arguments based on a variety of models [7–14] indicate that the transition as a function of  $T$  is first order at large  $\mu$ .<sup>2</sup> This suggests that the phase diagram features a critical point  $E$  at which the line of first order phase transitions present for  $\mu > \mu_E$  ends, as shown in Figure 1.<sup>3</sup> At  $\mu_E$ , the phase transition at  $T = T_E$  is second order and is in the Ising universality class [11,12]. Although the pions remain massive, the correlation length in the  $\sigma$  channel diverges due to universal long wavelength fluctuations of the order parameter. This results in characteristic signatures, analogues of critical opalescence in the sense that they are unique to collisions which freeze out near the critical point, which can be used to discover  $E$  [1,2].

The position of the critical point is, of course, not universal. Furthermore, it is sensitive to the value of the strange quark mass.  $\mu_E$  decreases as  $m_s$  is decreased [1], and at some  $m_s^c$ , it reaches  $\mu_E = 0$  and the transition becomes entirely first order [15]. The value of  $m_s^c$  is an open question, but lattice simulations suggest that it is about half the physical strange quark mass [16,17], although these results are not yet conclusive [18]. Of course,

<sup>2</sup>The realization that this phase transition is not simply a transition at which the chiral condensate vanishes, but is instead a transition at which a phase with nonzero chiral condensate and a phase with nonzero superconducting condensate compete and coexist has further strengthened the arguments in favor of a first order phase transition on the  $\mu$ -axis in two-flavor QCD [9,11,13,14].

<sup>3</sup>If the up and down quarks were massless,  $E$  would be a tricritical point, at which the first order transition becomes second order.

experimentalists cannot vary  $m_s$ . They can, however, vary  $\mu$ . The AGS, with beam energy 11 AGeV corresponding to  $\sqrt{s} = 5$  GeV, creates fireballs which freeze out near  $\mu \sim 500 - 600$  MeV [19]. When the SPS runs with  $\sqrt{s} = 17$  GeV (beam energy 158 AGeV), it creates fireballs which freeze out near  $\mu \sim 200$  MeV [19]. RHIC will make even smaller values of  $\mu$  accessible. By dialing  $\sqrt{s}$  and thus  $\mu$ , experimenters can find the critical point  $E$ . Finding  $E$  would confirm that  $m_s^c$  is less than the physical value of  $m_s$ , and that the phase transition at lower chemical potentials ( $\mu < \mu_E$ ) is a smooth crossover.

## 2. Discovering the Critical Point

Predicting  $\mu_E$ , and thus suggesting the  $\sqrt{s}$  to use to find  $E$ , is beyond the reach of present theoretical methods because  $\mu_E$  is both nonuniversal and sensitively dependent on the mass of the strange quark. Crude models suggest that  $\mu_E$  could be  $\sim 600 - 800$  MeV in the absence of the strange quark [11,12]; this in turn suggests that in nature  $\mu_E$  may have of order half this value, and may therefore be accessible at the SPS if the SPS runs with  $\sqrt{s} < 17$  GeV. However, at present theorists cannot predict the value of  $\mu_E$  even to within a factor of two. The SPS can search a significant fraction of the parameter space; if it does not find  $E$ , it will then be up to the RHIC experiments to map the  $\mu_E < 200$  MeV region.

Locating  $E$  on the phase diagram can only be done convincingly by an experimental discovery. Theorists can, however, do reasonably well at describing the phenomena that occur near  $E$ , thus enabling experimenters to locate it. This is the goal of Ref. [2]. The signatures proposed there are based on the fact that  $E$  is a genuine thermodynamic singularity at which susceptibilities diverge and the order parameter fluctuates on long wavelengths. The resulting signatures are *nonmonotonic* as a function of  $\sqrt{s}$ : as this control parameter is varied, we should see the signatures strengthen and then weaken again as the critical point is approached and then passed.

The simplest observables to use are the event-by-event fluctuations of the mean transverse momentum of the charged particles in an event,  $p_T$ , and of the total charged multiplicity in an event,  $N$ . The fluctuations observed in SPS collisions by NA49 are as perfect gaussians as the data statistics allow, as expected for freeze-out from a system in thermal equilibrium [20]. The data on multiplicity fluctuations show evidence for a nonthermodynamic contribution, which is to be expected since the extensive quantity  $N$  is sensitive to the initial size of the system and thus to nonthermodynamic effects like variation in impact parameter. The contribution of such effects to the fluctuations have now been estimated [21,22]; the combined thermodynamic and nonthermodynamic fluctuations are in satisfactory agreement with the data [22]. The width of the event-by-event distribution of the mean  $p_T$  of the charged pions in a single event<sup>4</sup> is in good agreement with predictions based on noncritical thermodynamic fluctuations [2]. That is, NA49 data are consistent with the hypothesis that almost all the observed event-by-event fluctuation in mean  $p_T$ , an intensive quantity, is thermodynamic in origin. This bodes well for the detectability of systematic changes in thermodynamic fluctuations near  $E$ .

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<sup>4</sup>This width can be measured even if one observes only two pions per event [23]; large acceptance data as from NA49 is required in order to learn that the distribution is gaussian, that thermodynamic predictions may be valid, and that the width is therefore the only interesting quantity to measure.

One analysis described in detail in Ref. [2] is based on the ratio of the width of the true event-by-event distribution of the mean  $p_T$  to the width of the distribution in a sample of mixed events. This ratio was called  $\sqrt{F}$ . NA49 has measured  $\sqrt{F} = 1.002 \pm 0.002$  [20,2], which is consistent with expectations for noncritical thermodynamic fluctuations.<sup>5</sup> Critical fluctuations of the  $\sigma$  field, i.e. the characteristic long wavelength fluctuations of the order parameter near  $E$ , influence pion momenta via the (large)  $\sigma\pi\pi$  coupling and increase  $\sqrt{F}$  [2]. The effect is proportional to  $\xi_{\text{freezeout}}^2$ , where  $\xi_{\text{freezeout}}$  is the  $\sigma$ -field correlation length of the long-wavelength fluctuations at freezeout [2]. If  $\xi_{\text{freezeout}} \sim 3$  fm, the ratio  $\sqrt{F}$  increases by  $\sim 3-5\%$ , ten to twenty times the statistical error in the present measurement [2]. This observable is valuable because data on it has been analyzed and presented by NA49, and it can therefore be used to learn that Pb+Pb collisions at 158 AGeV do *not* freeze out near  $E$ .

Once  $E$  is located, however, other observables which are more sensitive to critical effects will be more useful. For example, a  $\sqrt{F_{\text{soft}}}$ , defined using only the softest 10% of the pions in each event, will be much more sensitive to the critical long wavelength fluctuations. The higher  $p_T$  pions are less affected by the  $\sigma$  fluctuations [2], and these relatively unaffected pions dominate the mean  $p_T$  of all the pions in the event. This is why the increase in  $\sqrt{F}$  near the critical point will be much less than that of  $\sqrt{F_{\text{soft}}}$ .

The multiplicity of soft pions is an example of an observable which may be used to detect the critical fluctuations without an event-by-event analysis. The post-freezeout decay of sigma mesons, which are copious and light at freezeout near  $E$  and which decay subsequently when their mass increases above twice the pion mass, should result in a population of pions with  $p_T \sim m_\pi/2$  which appears only for freezeout near the critical point [2]. If  $\xi_{\text{freezeout}} > 1/m_\pi$ , this population of unusually low momentum pions will be comparable in number to that of the “direct” pions (i.e. those which were pions at freezeout) and will result in a large signature.

The variety of observables which should *all* vary nonmonotonically with  $\sqrt{s}$  (and should all peak at the same  $\sqrt{s}$ ) is sufficiently great that if it were to turn out that  $\mu_E < 200$  MeV, making  $E$  inaccessible to the SPS, all four RHIC experiments could play a role in the study of the critical point.

### 3. Slowing Out of Equilibrium: How Large Can $\xi$ Grow?

The purpose of Ref. [25] is to estimate how large  $\xi_{\text{freezeout}}$  can become, thus making the predictions of Ref. [2] for the magnitude of various signatures more quantitative. In an ideal system of infinite size which was held at  $T = T_E$ ;  $\mu = \mu_E$  for an infinite time, the correlation length  $\xi$  would be infinite. Ref. [2] estimated that finite size effects limit  $\xi$  to be about 6 fm at most. We argue in Ref. [25] that limitations imposed by the finite duration of a heavy ion collision are more severe, preventing  $\xi$  from growing larger than about  $2/T_E \sim 3$  fm.

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<sup>5</sup>In an infinite system made of classical particles which is in thermal equilibrium,  $\sqrt{F} = 1$ . Bose effects increase  $\sqrt{F}$  by  $1 - 2\%$  [24,2]; an anticorrelation introduced by energy conservation in a finite system — when one mode fluctuates up it is more likely for other modes to fluctuate down — decreases  $\sqrt{F}$  by  $1 - 2\%$  [2]; two-track resolution also decreases  $\sqrt{F}$  by  $1 - 2\%$  [20]. The contributions due to correlations introduced by resonance decays and due to fluctuations in the flow velocity are each much smaller than 1% [2].

The nonequilibrium dynamics analyzed in Ref. [25] is *guaranteed* to occur in a heavy ion collision which passes near  $E$ , even if local thermal equilibrium is achieved at a higher temperature during the earlier evolution of the plasma created in the collision. We assume early thermal (although not necessarily chemical) equilibration, and ask how the system evolves out of equilibrium as it passes  $E$ . For the present, assume that the system cools through the critical point  $E$  as sketched in trajectory (b) of Figure 1. If it were to cool arbitrarily slowly,  $\xi(T) = \xi_{\text{eq}}(T)$  would be maintained at all temperatures, and  $\xi$  would diverge at  $T_E$ . However, it would take an infinite time for  $\xi$  to grow infinitely large. Indeed, near a critical point, the longer the correlation length, the longer the equilibration time, and the slower the correlation length can grow. This critical slowing down means that the correlation length cannot grow as fast as  $\xi_{\text{eq}}$ , and the system cannot stay in equilibrium.

We use the theory of dynamical critical phenomena to describe the effects of critical slowing down on the time development of the correlation length  $\xi(t)$ , using the three-dimensional Ising model to describe  $\xi_{\text{eq}}(T, \mu)$  near  $E$  [25]. In the Ising model, the order parameter and the correlation length are functions of the reduced temperature  $r$  and the magnetic field  $h$ . (In the Ising model,  $r$  is defined as  $(T - T_c)/T_c$  and is usually called  $t$ ; we reserve the symbol  $t$  for time, however.) We assume that the  $r$ - and  $h$ -axes are as sketched in Figure 1, although in reality they will be deformed. We consider cooling along trajectories that pass through  $E$  at various angles, like (a) and (b) sketched in Figure 1 and also consider trajectories like (c) which come near to but miss  $E$ . Our results depend on the universal function describing  $\xi_{\text{eq}}(r, h)$ , on the universal dynamical exponent  $z$  describing critical slowing down (perturbations away from equilibrium relax toward equilibrium on a timescale which scales with  $\xi$  like  $A\xi^z$  [26]), on the nonuniversal constant  $A$ , the nonuniversal constants which relate  $r$  and  $h$  to  $(T - T_E)$  and  $(\mu - \mu_E)$ , on  $T_E$  which we take to be  $\sim 140$  MeV, and finally on the cooling rate  $|dT/dt|$  which we estimate to be 4 MeV/fm [27–29,25].

Our results have a number of consequences, detailed in the remainder of this section, which should be taken into account both in planning experimental searches for the QCD critical point, and in planning future theoretical work.

We do indeed find that because of the critical slowing down of the long wavelength dynamics near  $E$ , the correlation length does not have time to grow as large as it would in equilibrium: we find  $\xi_{\text{freezeout}} \sim 2/T_E \sim 3$  fm for trajectories passing near  $E$ .

Although critical slowing down hinders the growth of  $\xi$ , it also slows the decrease of  $\xi$  as the system continues to cool below the critical point. As a result,  $\xi$  does not decrease significantly between the phase transition and freezeout.

Our estimate that  $\xi$  does not grow larger than  $2/T_E$  is robust in three senses. First, it depends very little on the angle with which the trajectory passes through  $E$ . Second, it turns out to depend on only one combination of all the nonuniversal quantities which play a role. We call this parameter  $a$ ; it is proportional to  $|dT/dt|^{-1}$ . Third, our results do not depend sensitively on  $a$ . We show that the maximum value of  $\xi$  scales like  $a^{\frac{\nu/\beta\delta}{1+z\nu/\beta\delta}} \approx a^{0.215}$  [25].<sup>6</sup> Thus, for example,  $|dT/dt|$  would have to be a factor of 25 smaller than we estimate

<sup>6</sup>A scaling law of this form (but of course with different exponents) relating the maximum correlation length which is reached to the cooling rate with which a second order phase transition is traversed was first discovered in the theory of defect formation at a second order phase transition [30]. In this context, the maximum correlation length reached during the phase transition sets the scale for the initial separation

in order for  $\xi$  to grow to  $4/T_E$  instead of  $2/T_E$ . Although our results are robust in this sense, they cannot be treated as precise because our assumption that the dynamics of  $\xi$  in QCD is described by the universal classical dynamics of the three-dimensional Ising model only becomes precise if  $\xi \gg 1/T_E$ , while our central result is that  $\xi$  does not grow beyond  $\sim 2/T_E$ . A 3+1-dimensional quantum field theoretical treatment of the interplay between cooling and the dynamics of critical slowing down is not yet available, but promising first steps in this direction can be found in Ref. [34].

A result which is of great importance in the planning of experimental searches is that one need not hit  $E$  precisely in order to find it. Our analysis of trajectories like (c) of Figure 1 demonstrates that if one were to do a scan with collisions at many finely spaced values of the energy and thus  $\mu$ , one would see signatures of  $E$  with approximately the same magnitude over a broad range of  $\mu$ . The magnitude of the signatures will not be narrowly peaked as  $\mu$  is varied. As long as one gets close enough to  $E$  that the equilibrium correlation length is  $(2 - 3)/T_E$ , the actual correlation length  $\xi$  will grow to  $\sim 2/T_E$ . There is no advantage to getting closer to  $E$ , because critical slowing down prevents  $\xi$  from getting much larger even if  $\xi_{\text{eq}}$  does. Data at many finely spaced values of  $\mu$  is *not* called for.<sup>7</sup>

Knowing that we are looking for  $\xi_{\text{freezeout}} \approx 3$  fm is very helpful in suggesting how to employ the signatures described in detail in Ref. [2]. The excess of pions with  $p_T \sim m_\pi/2$  arising from post-freezeout decay of sigmas is large as long as  $\xi_{\text{freezeout}} \sim 1/m_\pi$ , and does not increase much further if  $\xi_{\text{freezeout}}$  is longer. This makes it an ideal signature. The increase in the event-by-event fluctuations in the mean transverse momentum of the charged pions in an event (described by the ratio  $\sqrt{F}$  of Ref. [2]) is proportional to  $\xi_{\text{freezeout}}^2$ . The results of Ref. [2] suggest that for  $\xi_{\text{freezeout}} \sim 3$  fm, this will be a 3 – 5% effect. This is ten to twenty times larger than the statistical error in the present NA49 data, but not so large as to make one confident of using this alone as a signature for  $E$ . The solution is to use signatures which focus on the event-by-event fluctuations of only the low momentum pions. Unusual event-by-event fluctuations in the pion momenta arise via the coupling between the pions and the sigma order parameter which, at freezeout, is fluctuating with correlation length  $\xi_{\text{freezeout}}$ . This interaction has the largest effect on the softest pions [2].  $\sqrt{F_{\text{soft}}}$ , described in the previous section, is a good example of an observable which takes advantage of this. Depending on the details of the cuts used to define it, it should be enhanced by many tens of percent in collisions passing near  $E$ . Ref. [2] suggests other such observables, and more can surely be found. Together, the excess multiplicity at low momentum (due to post-freezeout sigma decays) and the excess event-by-event fluctuation of the momenta of the low momentum pions (due to their coupling to

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between defects — vortices in a superfluid or in liquid crystals, for example — created as the system cools through the transition. This initial network of defects coarsens at later times. The Ising phase transition of interest to us creates no defects. It is nevertheless very pleasing that scaling laws analogous to the one we need have been tested quantitatively in numerical simulations of defect formation and dynamics [31] and, furthermore, are supported by data from experiments on liquid crystals [32] and superfluid  $^3\text{He}$  [33].  
<sup>7</sup>Analysis within the toy model of Ref. [11] suggests that in the absence of the strange quark, the range of  $\mu$  over which  $\xi_{\text{eq}} > 2$  fm is about  $\Delta\mu \sim 120$  MeV for  $\mu_E \sim 800$  MeV. Similar results can be obtained [35] within a random matrix model [12]. It is likely over-optimistic to estimate  $\Delta\mu \sim 120$  MeV when the effects of the strange quark are included and  $\mu_E$  itself is reduced. A conservative estimate would be to use the models to estimate that  $\Delta\mu/\mu_E \sim 15\%$ .

the order parameter which is fluctuating with correlation length  $\xi_{\text{freezeout}}$ ) should allow a convincing detection of the critical point  $E$ . Both should behave nonmonotonically as the collision energy, and hence  $\mu$ , are varied. Both should peak for those heavy ion collisions which freeze out near  $E$ , with  $\xi_{\text{freezeout}} \sim 3$  fm.

#### 4. Quenching Out of Equilibrium vs. Slowing Out of Equilibrium vs. Bubbling Out of Equilibrium

As the matter created in a heavy ion collision cools through the QCD phase transition, it may be driven out of equilibrium. The way in which this may happen is quite different depending on whether the phase transition is traversed with  $\mu \gg \mu_E$ ,  $\mu \ll \mu_E$  or  $\mu \sim \mu_E$ .

If  $\mu > \mu_E$ , the phase transition is first order; if  $(\mu - \mu_E)$  is large enough, the transition may be sufficiently strongly first order that it proceeds via the nucleation of well-separated bubbles of hadronic matter, which then grow. This bubbling results in a spatially inhomogeneous hadronic system. If the system freezes out before it gets re-homogenized, the result will be large, non-gaussian fluctuations in hadron multiplicity as a function of rapidity [36]. These overdense and underdense regions in rapidity would provide a distinctive signature, and have not been seen.

If  $\mu \ll \mu_E$ , the phase transition is a smooth crossover. This is the region of the diagram where the transition may be traversed most quickly, with the most rapid cooling rate. (For  $\mu \sim \mu_E$ , the specific heat is unusually large and therefore even though the rate of decrease of energy density is not unusual, the cooling rate  $|dT/dt|$  is unusually small. For  $\mu \gg \mu_E$ , neither of the two phases which coexist at the first order phase transition has an unusually large specific heat. The system nevertheless spends a long time at the coexistence temperature  $T_c$  because of the release of latent heat. Thus,  $|dT/dt|$  is allowed to be largest for  $\mu \ll \mu_E$ .) Although the smooth crossover with nondivergent specific heat allows a large  $|dT/dt|$  in principle, the cooling rate which is actually achieved will depend on the system size and on the duration of time between the initial collision and the phase transition. If conditions can be found in which  $|dT/dt|$  can be made large enough, the dynamics may then be well-modelled as a quench, in which most of the degrees of freedom in the system are imagined to cool arbitrarily rapidly. In this circumstance, the long wavelength modes of the pion field become unstable to exponential growth [37], leading to the formation of long wavelength pion oscillations. Once excited, these oscillations may be further amplified by parametric resonance [38].<sup>8</sup> These long wavelength disorientations of the chiral condensate result in characteristic fluctuations in the ratio of the number of low  $p_T$  neutral pions to the number of low  $p_T$  charged pions [39,37,40]. Other signatures of DCC oscillations have also been discussed [41]. WA98 [42] and NA49 [20] have looked for signatures of DCC oscillations in 158 AGeV PbPb collisions, but none have been seen.

The considerations above suggest that RHIC collisions provide more favorable conditions for DCC production than SPS collisions, because  $\mu$  is smaller and one is more likely to have a smooth crossover which allows large  $|dT/dt|$ . However, the hadronic systems produced in RHIC collisions will be large, will traverse the phase transition late, and will freeze out even later. The late transition reduces the cooling rate, making DCC

<sup>8</sup>As seems indicated [38] by the long period of time over which amplification is seen in the simulations of Ref. [37].

formation less likely, and the late freeze out makes wash-out of DCC signatures a worry. This suggests that the most favorable conditions in which to look for DCC signatures may be RHIC collisions of medium-sized ions, or AuAu collisions at RHIC with impact parameters of order the nuclear radius. (That is, midway between central and peripheral collisions.) Quite different considerations have lead Asakawa, Minakata and Müller to propose that non-central RHIC collisions provide the most favorable conditions for DCC formation [43]: these collisions feature strong electromagnetic  $\mathbf{E} \cdot \mathbf{B}$  fields which may, through the anomaly, result in a coherent excitation of the  $\pi^0$  field. There are therefore three reasons to suggest that non-central RHIC collisions may yield DCC signatures: the smooth crossover and small system size allow the system to cool through the transition faster, favoring the instability driven growth of DCCs; the electromagnetic kick may further drive their growth; and, the smaller system size will hasten freezeout. The first and third reasons also apply to RHIC collisions of medium-sized ions.

For  $\mu \sim \mu_E$ , the specific heat is larger in the vicinity of the critical point than anywhere else on the phase diagram, and the cooling rate  $|dT/dt|$  is correspondingly slow. This slow cooling means that those collisions which create matter which cools through the transition region with  $\mu \sim \mu_E$  are *least* likely to be driven far from equilibrium. That is, we do not expect DCC signatures or signatures of bubbling in these collisions. The interesting twist described in the previous section is that it is nevertheless true that such collisions cannot stay *precisely* in equilibrium [25]. Even though the system cools through the vicinity of  $E$  slowly, the correlation length still grows much too slowly to become as long as it would in equilibrium. We have discussed this phenomenon of slowing out of equilibrium at length, as it determines the magnitude of signatures of the critical point. Here, we note that this non-equilibrium physics is much milder than quenching or bubbling. Quenching or bubbling create *more* fluctuations than would be obtained in thermal equilibrium. They also create fluctuations which are non-gaussian in characteristic ways. In contrast, slowing out of equilibrium serves to *reduce* the growth of fluctuations relative to what would be achieved if equilibrium were maintained. The critical point  $E$  will be signalled by an increase in the magnitude of suitably chosen event-by-event fluctuations, for example parametrized by the growth in  $\sqrt{F}$  and  $\sqrt{F_{\text{soft}}}$ , but these fluctuations are expected to remain gaussian. Slowing out of equilibrium means that these gaussian fluctuations are smaller than they would be if equilibrium were maintained near  $E$ . They are nevertheless larger than the equilibrium thermal fluctuations far from  $E$  and hence result in nonmonotonic signatures which peak for collisions which freezeout near  $E$ .<sup>9</sup>

We have learned much from the beautiful gaussian event-by-event fluctuations observed by NA49. The magnitude of these fluctuations are consistent with the hypothesis that the hadronic system at freezeout is in approximate thermal equilibrium. These and other data show no signs that the system at freezeout has recently been bubbled or quenched. There is also no sign of the larger, but still gaussian, fluctuations which would signal freezeout near the critical point  $E$ . Combining these observations with the observation

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<sup>9</sup>In addition to restricting the growth of fluctuations, critical slowing down retards the decrease of the fluctuations after the transition. In addition, the large specific heat and consequent slow cooling rate result in an elevated freezeout temperature [2]. This suggests that signatures of  $E$  will not be washed out before freezeout; if necessary, however, smaller ions or non-central collisions can be used to further elevate the freezeout temperature [44,2].



of several tantalizing indications that the matter created in SPS collisions is not well described at early times by hadronic models [45] suggests that collisions at the SPS may be exploring the crossover region to the left of the critical point  $E$ , in which the matter is not well-described as a hadron gas but is also not well-described as a quark-gluon plasma. This speculation could be confirmed in two ways. First, if the SPS is probing the crossover region then the coming experiments at RHIC may discover direct signatures of an early partonic phase, which are well-described by theoretical calculations beginning from an equilibrated quark-gluon plasma. Second, if 158 AGeV collisions are probing the crossover region not far to the left of the critical point  $E$ , then running at lower energies would result in the discovery of  $E$ . If, instead, RHIC were to discover  $E$  with  $\mu_E < 200$  MeV, that would indicate that the SPS experiments have probed the weakly first order region just to the right of  $E$ . Regardless, discovering  $E$  would take all the speculation out of mapping this part of the QCD phase diagram.

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